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A CONTOUR FORMULA FOR COMPENSATED MICROSTRIP STEPS AND OPEN ENDS

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ABSTRACT

A formula for the contour of compensated microstrip steps and open ends is derived. Suitable for automated mask design, the expression generates an exponential transition minimizing the parasitic shunt capacitance and series inductance of microstrip step discontinuities.

INTRODUCTION

The parasitic reactances associated with microstrip steps and open ends have been investigated by many researchers. In the design of microstrip circuits, these parasitic elements must be accounted for. While the end effect is easily compensated by foreshortening, the presence of step parasitics complicates the design procedure, particularly in the case of broadband components.

The purpose of this paper is to present an expression which enables the designer to minimize if not eliminate the step parasitics at the *manufacturing* level so that they need not be considered at the *design* stage.

The approach follows the procedure described by Malherbe and Steyn [1] and Larsson [2] for the compensation of stripline steps: instead of changing suddenly in width, the wider strip is narrowed gradually in such a way that its characteristic impedance remains constant right up to the junction.

The open end may be considered as the limiting case of a step from finite to zero strip width. Both symmetrical and non-symmetrical steps will be treated in this paper.

Only recently, a geometrically very simple method of microstrip step compensation has been proposed by Chadha and Gupta [3] who chamfer the corners of symmetrical steps at an angle of 60^o. However, while this geometry seems to be valid only for a limited range of permittivities and impedance ratios, the approach chosen in the present paper is appropriate for any dielectric and any impedance ratio because it is based on accurate expressions for the characteristic impedance of microstrip by Hammerstad and Jensen [4].

DERIVATION OF THE CONTOUR FORMULA

Symmetrical Steps

Figure 1 shows a compensated symmetrical microstrip discontinuity. Compare the two segments ABCD and A'B'C'D' which are both Δx long. The contour y(x) must be determined in such a way that the total capacity of both segments is the same for all $x > 0$. For this purpose, the total capacity C_t of each segment is arbitrarily subdivided into two parts, namely the parallel plate capacity C_p and the fringe capacity C_f which is proportional to the length of the arc AB. Thus, for all $x > 0$:

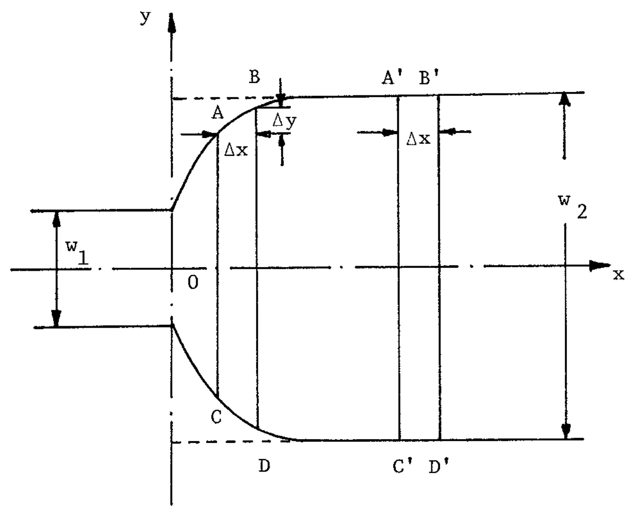

Fig. 1 Compensated symmetrical microstrip step discontinuity. The substrate thickness h is the same for all x and y.

$$C_t = C_p + C_f = C_p' + C_f' = C_t'. \quad (1)$$

$C_t = C_t'$, the total capacity of the segments, is related to the characteristic impedance and effective dielectric constant of a uniform strip of width w_2 as follows:

$$C_t = C_t' = \frac{\Delta x\sqrt{\varepsilon_e(w_2)}}{c_0 Z_0(w_2)} \quad (2)$$

where c_0 is the free-space velocity of light. $\varepsilon_e(w_2)$ and $Z_0(w_2)$ are the effective dielectric constant and the characteristic impedance of a microstrip line of width w_2 and height h. The values of the parallel plate capacities are:

(i) for the segment A'B'C'D':

$$C_p' = \varepsilon_0\varepsilon_r w_2\ \Delta x/h \quad (3a)$$

(ii) for the segment ABCD:

$$C_p = (\varepsilon_0\varepsilon_r/h)(2y\Delta x + \Delta x\Delta y) \quad (3b)$$

The values for the fringe capacities are:

(i) for the segment A'B'C'D':

$$C_f' = C_t' - C_p' \quad (4a)$$

(ii) for the segment ABCD:

$$C_f = \sqrt{(\Delta x)^2 + (\Delta y)^2}\ C_{f0} \quad (4b)$$

where C_{f0} is the fringe capacity per unit length of a

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uniform microstrip line of width 2y:

$$C_{f0} = \frac{\sqrt{\varepsilon_e(2y)}}{c_0 Z_0(2y)} - \varepsilon_0 \varepsilon_r 2y/h \qquad (5)$$

In the limit where $\Delta x \to dx$ and $\Delta y \to dy$, the equations (1) to (5) yield after some manipulation the following differential equation for the contour of the symmetrical microstrip step:

$$\frac{dy}{dx} = \pm\sqrt{\left\{\frac{\frac{\eta_o h}{2y\varepsilon_r}\,\frac{\varepsilon_e(w_2)}{Z_{01}(w_2)} - 1}{\frac{\eta_o h}{2y\varepsilon_r}\,\frac{\varepsilon_e(2y)}{Z_{01}(2y)} - 1}\right\}^2 - 1} \qquad (6)$$

where

η_o = 376.73 Ω

h = substrate thickness (distance between strip and ground plane)

ε_r = relative dielectric constant of substrate

$\varepsilon_e(w_2), \varepsilon_e(2y)$ = effective dielectric constant of a uniform microstrip line of width w_2 and 2y respectively on a substrate with permittivity ε_r.

$Z_{01}(w_2), Z_{01}(2y)$ = characteristic impedance of an *air-filled* microstrip line of width w_2 and 2y respectively.

Accurate *analytical* expressions for ε_e and Z_{01} have been published by Hammerstad and Jensen [4] and are repeated below for convenience.

The expression for the effective dielectric constant of microstrip is:

$$\varepsilon_e(u,\varepsilon_r) = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\,(1+10/u)^{-a(u)b(\varepsilon_r)} \qquad (7)$$

where

$$a(u) = 1 + \frac{1}{49}\ln\frac{u^4+(u/52)^2}{u^4+0.432} + \frac{1}{18.7}\ln\left[1+\left(\frac{u}{18.1}\right)^3\right] \qquad (8)$$

and

$$b(\varepsilon_r) = 0.564\left(\frac{\varepsilon_r-0.9}{\varepsilon_r+3}\right)^{0.053} \qquad (9)$$

The characteristic impedance of air-filled microstrip is:

$$Z_{01}(u) = \frac{\eta_o}{2\pi}\ln\left[\frac{f(u)}{u} + \sqrt{1+\left(\frac{2}{u}\right)^2}\right] \qquad (10)$$

where

$$f(u) = 6 + (2\pi-6)\exp\left[-\left(\frac{30.666}{u}\right)^{0.7528}\right] \qquad (11)$$

In these expressions, u is the width/height ratio ($u = w/h$), and for the purpose of evaluating (6) w becomes w_2 and 2y respectively. η_o in (10) is the impedance of free space (η_o = 376.73Ω).

It is also possible to include dispersion of ε_e in (6), but this has only a negligeably small effect on dy/dx.

Non-Symmetrical Steps

A non-symmetrical compensated microstrip step is shown in Fig. 2.

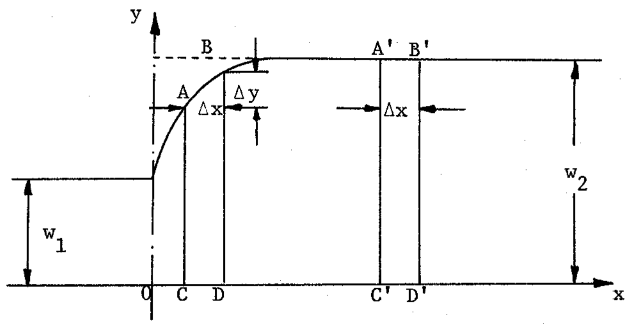

Fig. 2 Compensated non-symmetrical microstrip step discontinuity. The substrate thickness h is the same for all x and y

The parallel plate and fringe capacities of the segment A'B'C'D' are the same as for the symmetrical case and are given by (3a) and (4a) above.

For the segment ABCD we have:

$$C_p = (\varepsilon_o\varepsilon_r/h)(y\Delta x + \Delta x\Delta y/2) \qquad (12)$$

and

$$C_p = \left(\Delta x + \sqrt{(\Delta x)^2+(\Delta y)^2}\right) C_{fo}/2 \qquad (13)$$

where C_{fo} is the fringe capacity/unit length of a uniform microstrip line of width y:

$$C_{fo} = \frac{\sqrt{\varepsilon_e(y)}}{c_o Z_o(y)} - \varepsilon_o\varepsilon_r y/h \qquad (14)$$

In the limit where $\Delta x \to dx$ and $\Delta y \to dy$, the equations (1), (2) and (12) to (14) yield after some manipulation the following differential equation for the contour of the non-symmetrical microstrip step:

$$\frac{dy}{dx} = \sqrt{\left\{\frac{\frac{\eta_o h}{\varepsilon_r y}\left[\frac{2\varepsilon_e(w_2)}{Z_{01}(w_2)} - \frac{\varepsilon_e(y)}{Z_{01}(y)}\right] - 1}{\frac{\eta_o h}{\varepsilon_r y}\,\frac{\varepsilon_e(y)}{Z_{01}(y)} - 1}\right\}^2 - 1} \qquad (15)$$

where all quantities are interpreted as in (6) and in accordance with Fig. 2. ε_e and Z_{01} are given by (7) and (10) respectively.

GENERATION OF THE CONTOUR

It is both difficult and unnecessary to integrate (6) and (15) analytically. Instead, the contour can

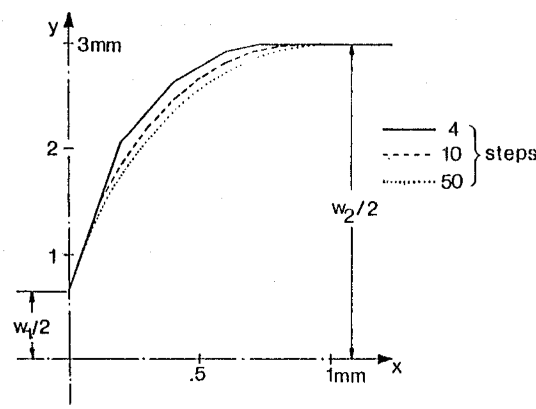

Fig. 3 Contour of compensated symmetrical microstrip step obtained with different step sizes dx. (w_1=1.25 mm, w_2=6 mm, h=0.508 mm, ε_r=2.22)

be generated in small successive steps of dx, starting at x = 0 where y corresponds to the width of the narrow strip. For each small step dx, (6) and (15) yield the corresponding increment dy until y reaches the width of the wider strip. While 10 steps approximate the contour quite well, a polygon consisting of 50 steps is virtually indistinguishable from the accurate contour (see Fig. 3)

MEASUREMENTS

In order to verify the validity of the contour formulas, the resonant frequencies of two strips of equal length ℓ were compared. One of them had square corners, while the other was rounded off to compensate the end effect (see Fig. 4). Both strips were located on the same substrate sufficiently far apart, and very loosely coupled in order to minimize errors due to variations in permittivity and to circuit loading. Resonance frequencies were measured in the transmission mode.

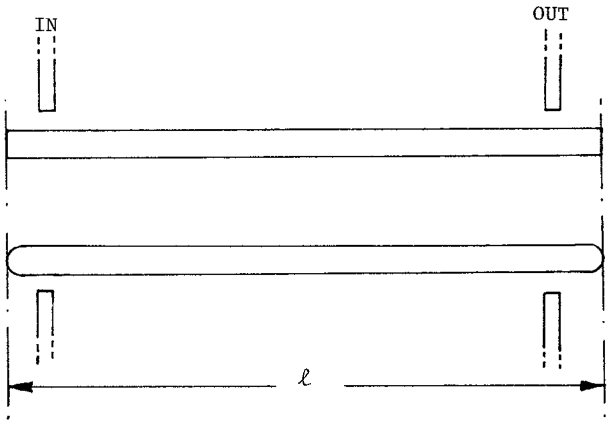

Fig. 4 Compensated and uncompensated resonant microstrips for the evaluation of the contour formula

Table 1 compares the results obtained for 10 cm long strips on RT/duroid ($\varepsilon_r = 2.22$) and on Epsilam 10 ($\varepsilon_r = 9.8$). It is clearly seen that the rounded strips resonate within 0.2% of the theoretical value, indicating that the end effect has effectively been reduced.

Material and Geometry	Resonant Frequencies (MHz)		
	Theoretical (no end effect)	Measured (Compensated)	Measured (Non-Compensated)
RT/Duroid, $\varepsilon_r = 2.22$	1052.45	1053.00	1044.81
h = 0.508 mm	2104.53	2106.02	2085.89
w = 5 mm, ℓ = 10 cm	3155.92	3152.68	3131.49
Epsilam 10, $\varepsilon_r = 9.8$	517.59	518.7	510.3
h = 0.635 mm	1034.56	1035.8	1023.0
w = 6.4 mm, ℓ = 10 cm	1550.32	1552.4	1535.1

Table 1 Comparison of theoretical and measured resonant frequencies of microstrip resonators with and without end compensation

DISCUSSION AND CONCLUSION

The parasitic reactances of step discontinuities in microstrip can be virtually eliminated by rounding off the corners of the wide strip. The contour is determined such that the characteristic impedance of the wide strip is constant right to the junction. This condition yields a differential contour equation which contains expressions for characteristic impedance and effective dielectric constant of microstrip. Its stepwise integration leads to a polygon approximation of the contour. The integration can be programmed on a pocket calculator or performed during automatic generation of the mask. This technique greatly reduces the complexity of circuit design. Its effectiveness has been demonstrated by measurements on microstrip resonators.

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